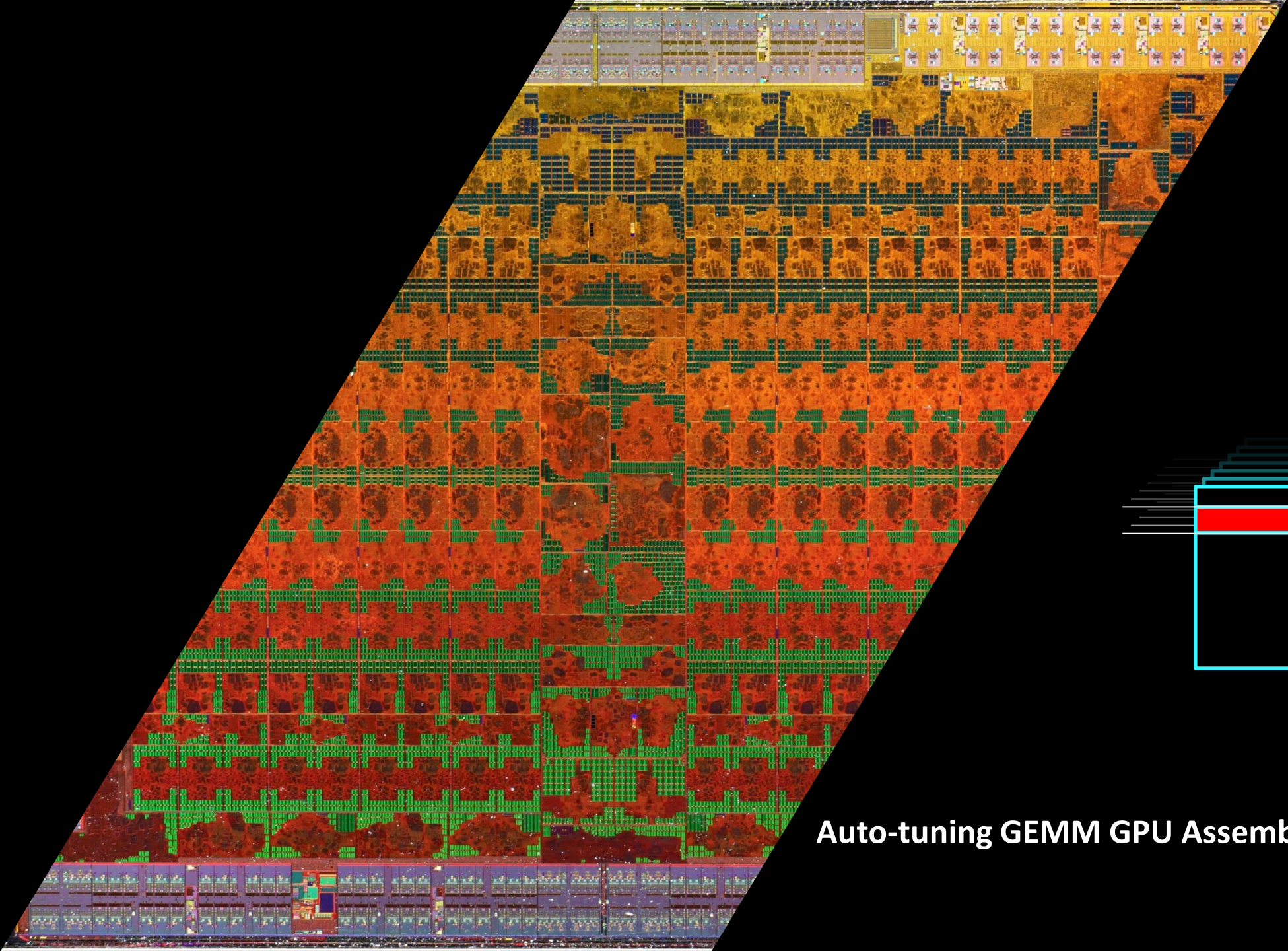
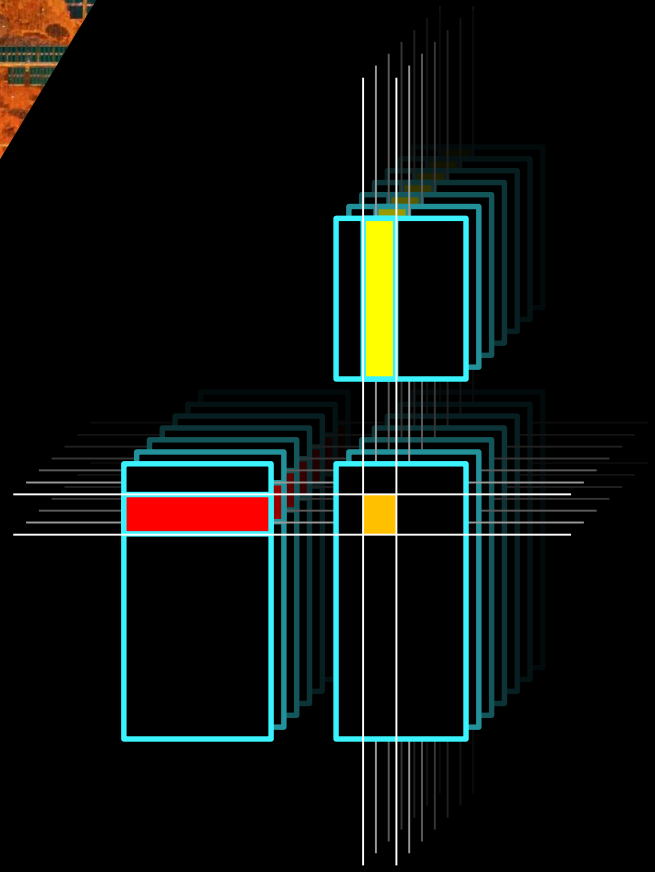


AMD



TENSILE

Auto-tuning GEMM GPU Assembly for All Problem Sizes

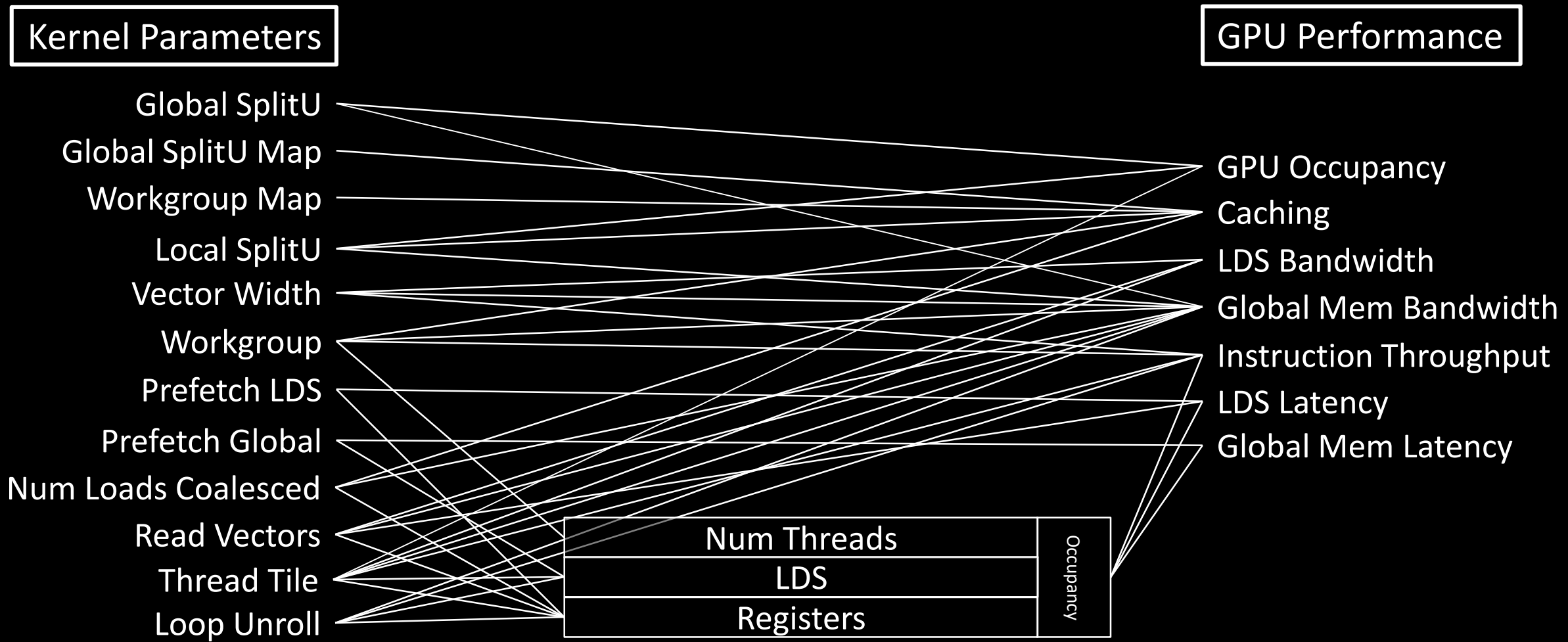
David E Tanner

MOTIVATION

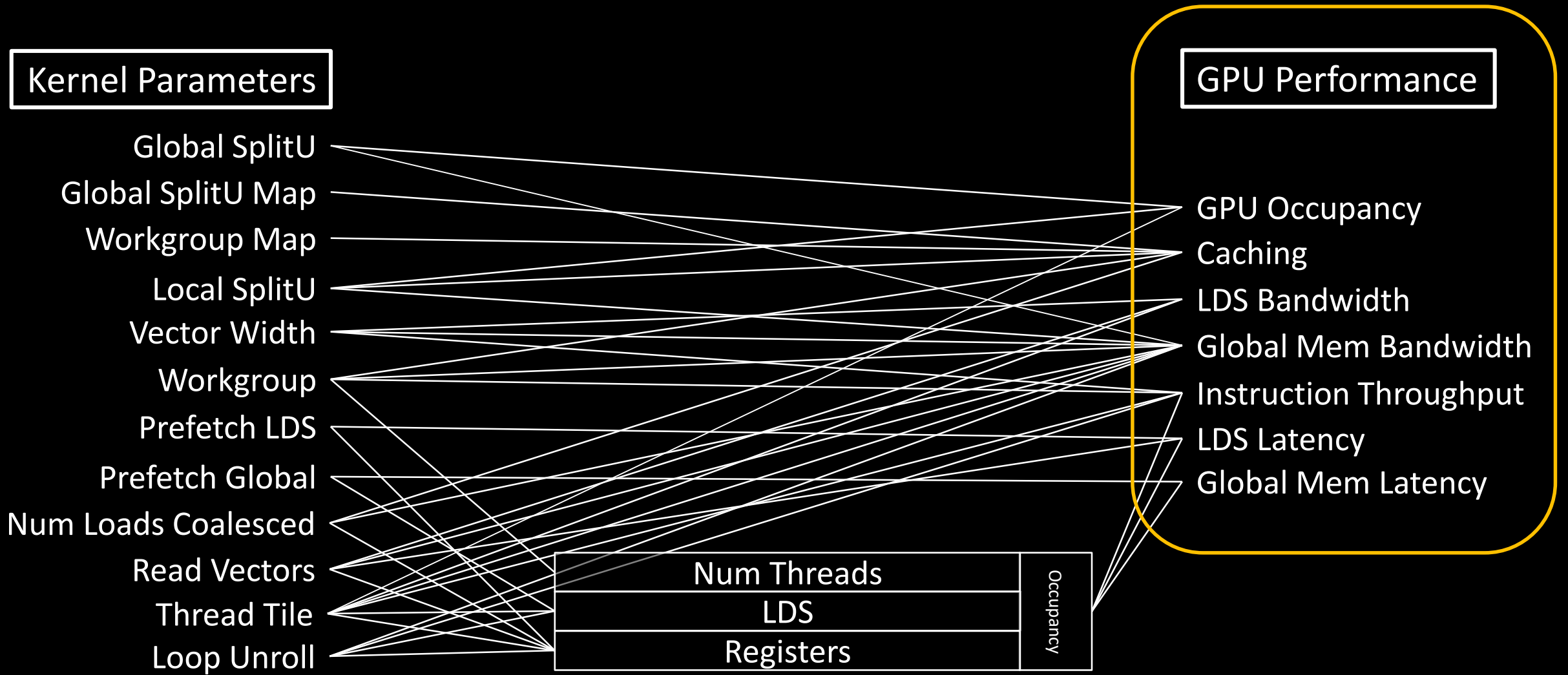
AMD

- GEMM API encapsulates trillions of problems, all of which behave differently on GPUs.
  - Precisions, Transpose A, Transpose B, M, N, K, Strides
- Many problem types behave similar to GEMM.
  - $C_{ij} = \sum_k A_{ik} * B_{kj}$ (gemm NN)
  - $C_{ijk} = \sum_l A_{ilk} * B_{jlk}$ (batched gemm NT)
  - $C_{ijk} = \sum_{lmn} A_{ilmnk} * B_{jlmnk}$ (batched gemm w/ 3D summation)
  - $C_{ijk} = \sum_{lmn} A_{inlkm} * B_{mjlkn}$ (batched gemm w/ 3D summation different data layout)
- Goal: auto-generate kernels with peak performance
  - For all problem types.
  - For all problem sizes.
  - On all GPUs.
  - Multiple languages: OpenCL, HIP, Assembly.

KERNEL PARAMETERS AFFECT GPU PERFORMANCE



KERNEL PARAMETERS AFFECT GPU PERFORMANCE



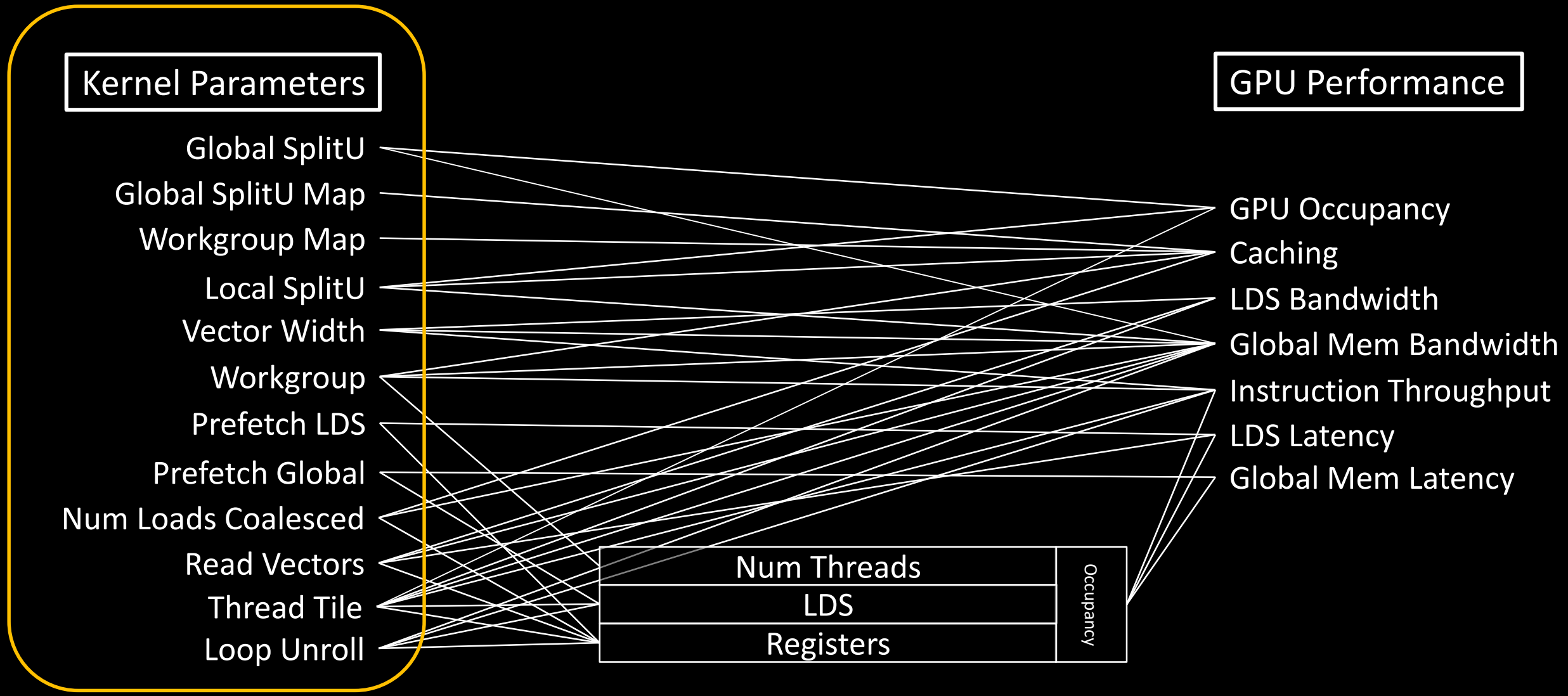
GPU PERFORMANCE PARAMETERS

VEGA10 FRONTIER EDITION

AMD

- Compute Throughput
  - 13.1 TFlops = 2 (flops/cycle)*64(CUs)*64(lanes/CU)*1600MHz
- LDS Bandwidth
  - TB/s
- Caches
  - L2 shared by all CUs
  - L1 dedicated to CU
- Global Memory Bandwidth
  - 480 GB/s
  - coalescing
- Global Memory Latency
  - hundreds of cycles
  - hide using CU-occupancy or ILP
- CU Occupancy
  - limited resources (VGPRs, LDS) per CU
- Whole-GPU Occupancy
  - hundreds of work-groups
- LDS Latency
  - tens of cycles
  - hide using CU-occupancy or ILP
- Instruction Divergence
  - all threads within workgroup do same instruction else need to compute and apply execution masks
- Instruction Throughput
  - gemm requires 2*M*N*K instructions, all extras hurt efficiency
  - minimize instructions which don’t count
  - maximize dual-issuing of instructions which don’t count with instructions that do; must be from different wavefronts
    - VALU, SALU, LDS, global memory, branch
- Power
  - VALU, LDS, memory, caches

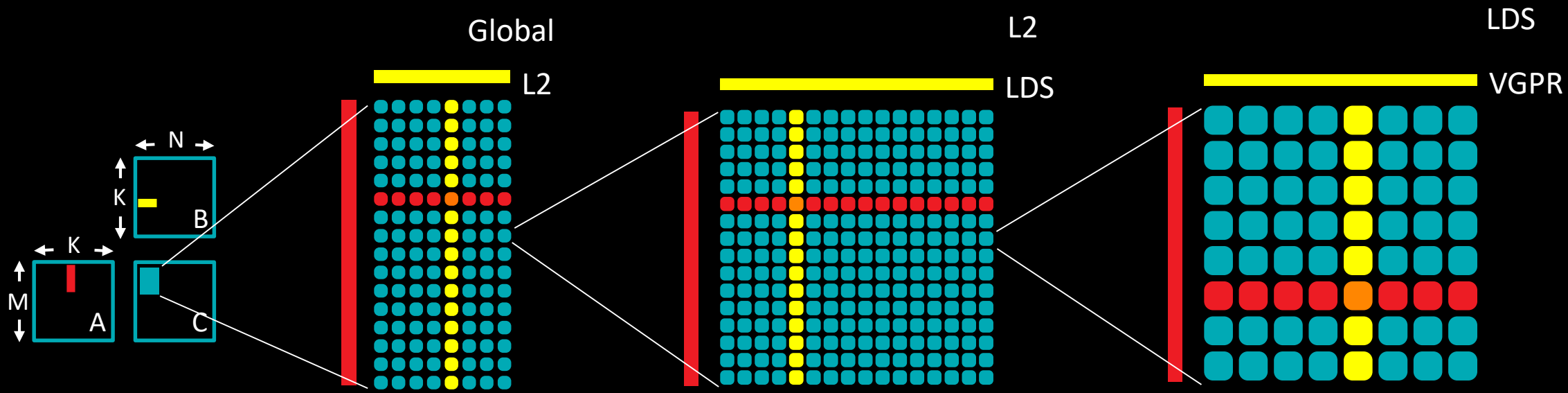
KERNEL PARAMETERS AFFECT GPU PERFORMANCE



GENERAL KERNEL-LEVEL STRATEGY

TILING

- Read more from higher bandwidth memory, less from lower bandwidth memory
- Larger tiles handle low bandwidth better, but create fewer workgroups



(0) GEMM

(1) Grid of 16x8 Workgroups

- Read from global once, store in L2
- Read from L2 and store into LDS 8-16x

(2) Workgroup of 16x16 Threads

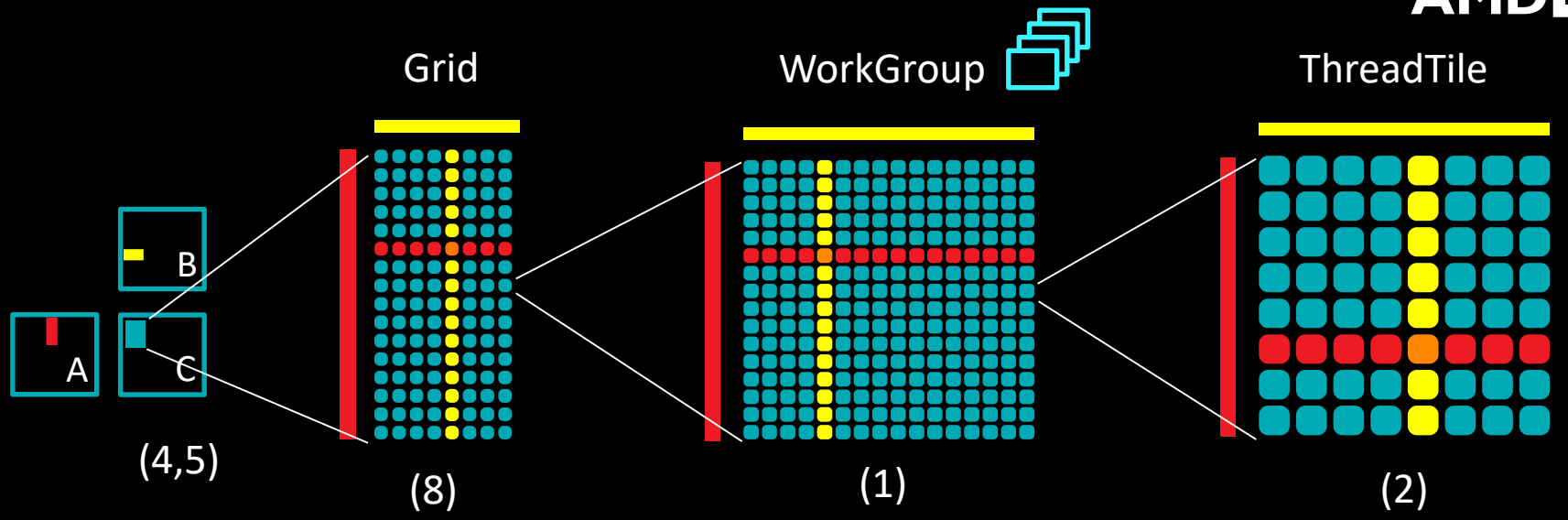
- Read from LDS and store in VGPRs 16x

(3) Thread has 8x8 elements

- 8 A VGPRs
- 8 B VGPRs
- 64 C VGPRs, 64 FMAs

KERNEL PARAMETERS

1) WorkGroup, LocalSplitU
2) ThreadTile
3) VectorWidth
4) GlobalSplitU
5) GlobalSplitUWGM
6) PrefetchGlobalRead
7) PrefetchLocalRead
8) WorkGroupMapping
9) LoopUnroll
10) NumLoadsCoalesced
11) GlobalReadCoalesceGroup
12) GlobalReadCoalesceVector
13) KernelLanguage
14) NonTemporal



(3,4,10,11,12)

(3,6)

(7)

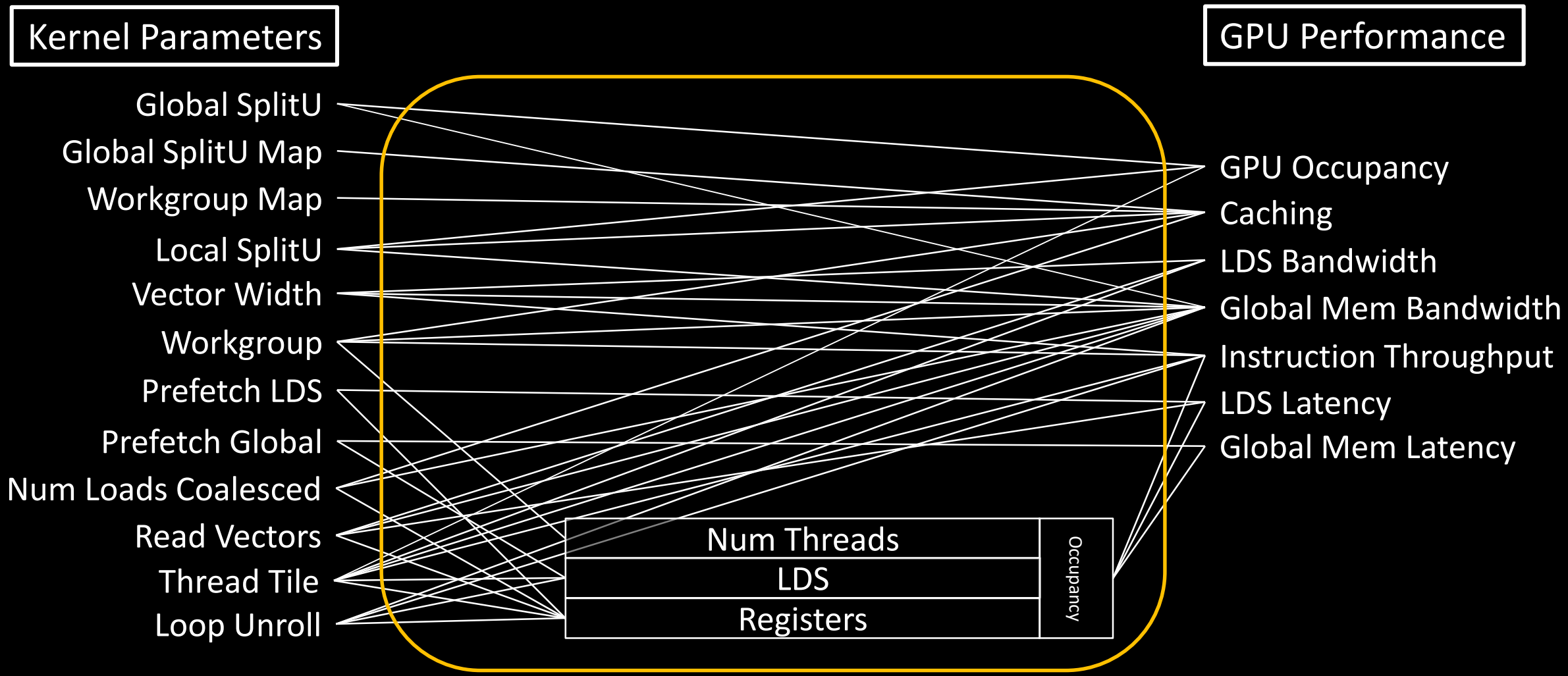
(9)

```
calculate addresses          read lds N-1
prefetch iter 0              wait global read
                             write lds
BEGIN LOOP                   swap lds red / black ptrs
load global iter 1           wait read lds N-2
read lds 1                   MACs N-2
wait lds read 0              wait write lds & N-1
MACs 0                       barrier
read lds 2                   read lds iter 1 subiter 0
wait lds read 1              MACs N-1
MACs 1                       END LOOP
…            (13)
                             write vgprs to C      (14)
```

KERNEL LANGUAGE

- Source Pros
  - Easy to write, debug.
  - Works on many GPUs.
  - Will work on future GPUs.
- Source Cons
  - Compiler doesn’t deliver peak performance.
    - Tuning for compiler and not architecture
  - Compiler regressions can lower performance.
- Assembly Pros
  - FAST!
  - Performance independent of compiler status.
- Assembly Cons
  - Hard to write.
  - Hard to maintain.
  - Bugs are catastrophic.

KERNEL PARAMETERS AFFECT GPU PERFORMANCE



SMOOTH PERFORMANCE

AMD

M \ N	16	32	64	112	176	256	352	464	592	736	896	1072	1264	1472	1696	1936	2192	2464	2752	3056	3376	3712	4064	4432	4816	5216	5632
16	29	62	118	205	314	448	596	735	890	1,024	1,131	1,204	1,401	1,502	1,562	1,625	1,646	1,750	1,754	1,823	1,833	1,887	1,937	1,890	1,976	1,977	2,036
32	60	116	238	406	615	880	1,137	1,424	1,722	2,006	2,271	2,304	2,628	2,877	2,953	3,120	3,171	3,365	3,452	3,617	3,619	3,654	3,800	3,710	3,848	3,896	3,992
64	121	236	452	774	1,168	1,635	2,030	2,402	2,785	3,173	3,539	3,625	4,206	4,506	4,723	4,959	4,936	5,146	5,483	5,846	5,644	5,979	6,404	5,869	6,251	6,449	6,510
112	206	409	790	1,270	1,831	2,365	2,758	3,275	3,590	3,989	4,353	4,398	5,002	5,487	5,422	6,033	5,625	5,791	6,362	6,879	6,652	6,891	7,436	7,058	7,203	7,016	7,458
176	316	633	1,201	1,848	2,404	3,178	3,457	4,012	4,506	4,695	5,479	5,397	6,223	5,894	6,204	6,908	6,755	7,237	7,001	7,332	7,864	7,630	8,156	7,868	7,704	8,343	8,023
256	456	878	1,575	2,342	3,065	4,021	4,271	4,847	5,052	5,810	5,780	5,968	6,579	6,898	6,811	7,593	7,095	7,458	7,936	8,597	8,012	8,558	9,211	8,357	8,761	8,387	8,835
352	604	1,186	2,141	2,971	3,566	4,566	4,720	5,458	5,924	6,001	6,555	6,683	7,363	7,298	7,224	7,899	7,811	7,934	7,886	8,573	8,636	8,741	9,013	8,483	8,534	9,188	8,854
464	755	1,447	2,489	3,340	4,113	5,100	5,480	5,877	5,810	6,906	7,229	7,075	7,789	7,932	7,927	8,217	8,232	8,403	8,562	8,950	8,526	8,708	9,437	8,882	9,007	8,905	9,378
592	947	1,711	2,892	3,749	4,510	5,561	6,022	5,942	6,393	7,288	7,512	7,550	7,734	8,013	8,120	8,403	8,677	8,430	8,848	9,118	8,947	8,898	9,573	9,414	9,861	9,461	9,707
736	1,073	2,018	3,415	4,254	4,966	6,241	6,099	6,967	7,254	7,666	7,913	8,111	8,476	8,693	8,424	9,010	8,796	9,114	9,079	9,678	9,715	9,801	10,293	9,534	9,505	10,182	9,853
896	1,228	2,225	3,755	4,450	5,394	6,442	6,447	6,794	6,950	7,527	7,974	8,695	8,841	8,860	9,428	9,395	9,785	9,516	9,655	9,758	10,327	10,173	10,338	10,442	10,216	10,456	10,786
1072	1,304	2,380	3,895	4,616	5,536	6,392	6,786	7,242	7,635	8,173	8,763	8,442	9,037	9,050	9,180	9,259	9,446	9,620	9,836	10,038	9,745	10,054	10,334	10,281	10,016	10,399	10,338
1264	1,427	2,716	4,490	5,222	6,367	7,212	7,401	7,782	7,755	8,474	9,031	9,012	9,125	9,373	9,506	9,739	9,929	9,614	10,071	10,300	10,133	10,140	11,079	10,544	11,055	10,580	11,158
1472	1,509	2,924	4,700	5,562	6,147	7,643	7,476	7,889	7,948	8,619	8,981	8,968	9,299	9,585	9,790	9,854	9,897	9,804	10,213	10,185	10,124	10,565	10,824	10,323	10,434	10,564	10,728
1696	1,608	3,128	4,896	5,594	6,562	7,673	7,503	8,015	8,196	8,409	9,484	9,100	9,521	9,792	9,641	9,605	9,943	9,926	10,056	10,052	10,381	10,304	10,753	10,358	10,234	10,841	10,593
1936	1,673	3,162	5,085	6,223	6,961	8,097	7,939	8,244	8,373	9,018	9,442	9,245	9,734	9,894	9,617	10,001	10,076	10,313	10,328	10,756	10,301	10,183	10,874	10,575	10,411	10,514	11,049
2192	1,666	3,291	5,152	6,069	7,078	8,025	7,870	8,285	8,692	8,773	9,876	9,419	9,929	9,894	9,946	10,081	9,841	10,083	10,276	10,515	10,485	10,543	10,970	10,806	10,726	10,663	10,651
2464	1,746	3,438	5,511	6,385	7,275	8,370	8,098	8,424	8,438	9,103	9,541	9,608	9,661	9,778	9,924	10,247	10,056	10,339	10,607	10,577	10,396	10,549	11,076	11,013	11,005	11,025	11,065
2752	1,805	3,501	5,703	6,353	7,109	8,311	8,062	8,530	8,825	9,046	9,846	9,765	10,032	10,185	10,053	10,301	10,260	10,617	10,726	10,485	10,647	11,294	11,298	10,608	10,669	10,732	10,868
3056	1,848	3,623	5,879	6,856	7,724	8,901	8,576	8,932	9,107	9,681	9,918	10,008	10,310	10,268	10,040	10,734	10,520	10,516	10,508	11,474	10,751	11,497	11,552	10,864	11,024	11,207	11,393
3376	1,836	3,570	5,910	6,907	7,934	8,733	8,671	8,510	8,930	9,739	10,468	9,701	10,125	10,145	10,398	10,296	10,481	10,394	10,669	10,722	10,744	10,801	10,989	11,199	10,775	11,037	11,290
3712	1,890	3,701	6,048	6,773	7,709	8,632	8,643	8,630	8,760	9,770	10,161	9,957	10,041	10,525	10,275	10,136	10,441	10,462	11,227	11,372	10,682	10,935	11,159	11,420	11,065	11,343	11,700
4064	1,948	3,824	6,511	7,413	8,252	9,440	8,982	9,380	9,553	10,282	10,555	10,289	11,005	10,828	10,748	10,806	10,915	11,081	11,257	11,483	10,935	11,228	11,521	11,210	11,548	11,328	11,685
4432	1,938	3,700	6,049	7,225	7,945	9,047	8,473	8,873	9,381	9,555	10,561	10,279	10,599	10,338	10,382	10,563	10,819	11,061	10,643	10,856	11,197	11,554	11,277	11,100	11,480	11,371	11,310
4816	1,966	3,819	6,334	7,244	7,879	9,041	8,608	9,070	9,868	9,534	10,245	9,994	11,052	10,466	10,211	10,420	10,701	11,044	10,694	11,023	10,786	11,199	11,605	11,480	11,416	11,375	11,396
5216	1,976	3,920	6,601	7,198	8,310	8,942	9,198	8,960	9,429	10,168	10,722	10,314	10,655	10,538	10,822	10,464	10,578	11,008	10,755	11,146	10,974	11,427	11,338	11,310	11,311	11,364	11,433
5632	2,034	3,996	6,180	7,169	7,800	9,051	8,528	9,295	9,440	9,765	10,740	10,183	10,998	10,577	10,521	10,906	10,524	10,999	10,797	11,250	11,160	11,654	11,602	11,166	11,244	11,355	11,513

K = 3104

GFlops

PREDICTED MAPPING

AMD

N

M	16	32	64	112	176	256	352	464	592	736	896	1072	1264	1472	1696	1936	2192	2464	2752	3056	3376	3712	4064	4432	4816	5216	5632
16	29	62	118	205	314	448	596	735	890	1,024	1,131	1,204	1,401	1,502	1,562	1,625	1,646	1,750	1,754	1,823	1,833	1,887	1,937	1,890	1,976	1,977	2,036
32	60	116	238	406	615	880	1,137	1,424	1,722	2,006	2,271	2,304	2,628	2,877	2,953	3,120	3,171	3,365	3,452	3,617	3,619	3,654	3,800	3,710	3,848	3,896	3,992
64	121	236	452	774	1,168	1,635	2,030	2,402	2,785	3,173	3,539	3,625	4,206	4,506	4,723	4,959	4,936	5,146	5,483	5,846	5,644	5,979	6,404	5,869	6,251	6,449	6,510
112	206	409	790	1,270	1,831	2,365	2,758	3,275	3,590	3,989	4,353	4,398	5,002	5,487	5,422	6,033	5,625	5,791	6,362	6,879	6,652	6,891	7,436	7,058	7,203	7,016	7,458
176	316	633	1,201	1,848	2,404	3,178	3,457	4,012	4,506	4,695	5,479	5,397	6,223	5,894	6,204	6,908	6,755	7,237	7,001	7,332	7,864	7,630	8,156	7,868	7,704	8,343	8,023
256	456	878	1,575	2,342	3,065	4,021	4,271	4,847	5,052	5,810	5,780	5,968	6,579	6,898	6,811	7,593	7,095	7,458	7,936	8,597	8,012	8,558	9,211	8,357	8,761	8,387	8,835
352	604	1,186	2,141	2,971	3,566	4,566	4,729	5,458	5,924	6,001	6,555	6,683	7,363	7,298	7,224	7,899	7,811	7,934	7,886	8,573	8,636	8,741	9,013	8,483	8,534	9,188	8,854
464	755	1,447	2,489	3,340	4,113	5,100	5,480	5,877	5,810	6,906	7,229	7,075	7,789	7,932	7,927	8,217	8,232	8,403	8,562	8,950	8,526	8,708	9,437	8,882	9,007	8,905	9,378
592	947	1,711	2,892	3,749	4,510	5,561	6,022	5,942	6,393	7,288	7,512	7,550	7,734	8,013	8,120	8,403	8,677	8,430	8,848	9,118	8,947	8,898	9,573	9,414	9,861	9,461	9,707
736	1,073	2,018	3,415	4,254	4,966	6,241	6,099	6,967	7,254	7,666	7,913	8,111	8,476	8,693	8,424	9,010	8,796	9,114	9,079	9,678	9,715	9,801	10,293	9,534	9,505	10,182	9,853
896	1,228	2,225	3,755	4,450	5,394	6,442	6,447	6,794	6,950	7,527	7,974	8,695	8,841	8,860	9,428	9,395	9,785	9,516	9,655	9,758	10,327	10,173	10,338	10,442	10,216	10,456	10,786
1072	1,304	2,380	3,895	4,616	5,536	6,392	6,786	7,242	7,635	8,173	8,763	8,442	9,037	9,050	9,180	9,259	9,446	9,620	9,836	10,038	9,745	10,054	10,334	10,281	10,016	10,399	10,338
1264	1,427	2,716	4,490	5,222	6,367	7,212	7,401	7,782	7,755	8,474	9,031	9,012	9,125	9,373	9,506	9,739	9,929	9,614	10,071	10,300	10,133	10,140	11,079	10,544	11,055	10,580	11,158
1472	1,509	2,924	4,700	5,562	6,147	7,643	7,476	7,889	7,948	8,619	8,981	8,968	9,299	9,585	9,790	9,854	9,897	9,804	10,213	10,185	10,124	10,565	10,824	10,323	10,434	10,564	10,728
1696	1,608	3,128	4,896	5,594	6,562	7,673	7,503	8,015	8,196	8,409	9,484	9,100	9,521	9,792	9,641	9,605	9,943	9,926	10,056	10,052	10,381	10,304	10,753	10,358	10,234	10,841	10,593
1936	1,673	3,162	5,085	6,223	6,961	8,097	7,939	8,244	8,373	9,018	9,442	9,245	9,734	9,894	9,617	10,001	10,076	10,313	10,328	10,756	10,301	10,183	10,874	10,575	10,411	10,514	11,049
2192	1,666	3,291	5,152	6,069	7,078	8,025	7,870	8,285	8,692	8,773	9,876	9,419	9,929	9,894	9,946	10,081	9,841	10,083	10,276	10,515	10,485	10,543	10,970	10,806	10,726	10,663	10,651
2464	1,746	3,438	5,511	6,385	7,275	8,370	8,098	8,424	8,438	9,103	9,541	9,608	9,661	9,778	9,924	10,247	10,056	10,339	10,607	10,577	10,396	10,549	11,076	11,013	11,005	11,025	11,065
2752	1,805	3,501	5,703	6,353	7,109	8,311	8,062	8,530	8,825	9,046	9,846	9,765	10,032	10,185	10,053	10,301	10,260	10,617	10,726	10,485	10,647	11,294	11,298	10,608	10,669	10,732	10,868
3056	1,848	3,623	5,879	6,856	7,724	8,901	8,576	8,932	9,107	9,681	9,918	10,008	10,310	10,268	10,040	10,734	10,520	10,516	10,508	11,474	10,751	11,497	11,552	10,864	11,024	11,207	11,393
3376	1,836	3,570	5,910	6,907	7,934	8,733	8,671	8,510	8,930	9,739	10,468	9,701	10,125	10,145	10,398	10,296	10,481	10,394	10,669	10,722	10,744	10,801	10,989	11,199	10,775	11,037	11,290
3712	1,890	3,701	6,048	6,773	7,709	8,632	8,643	8,630	8,760	9,770	10,161	9,957	10,041	10,525	10,275	10,136	10,441	10,462	11,227	11,372	10,682	10,935	11,159	11,420	11,065	11,343	11,700
4064	1,948	3,824	6,511	7,413	8,252	9,440	8,982	9,380	9,553	10,282	10,555	10,289	11,005	10,828	10,748	10,806	10,915	11,081	11,257	11,483	10,935	11,228	11,521	11,210	11,548	11,328	11,685
4432	1,938	3,700	6,049	7,225	7,945	9,047	8,473	8,873	9,381	9,555	10,561	10,279	10,599	10,338	10,382	10,563	10,819	11,061	10,643	10,856	11,197	11,554	11,277	11,100	11,480	11,371	11,310
4816	1,966	3,819	6,334	7,244	7,879	9,041	8,608	9,070	9,868	9,534	10,245	9,994	11,052	10,466	10,211	10,420	10,701	11,044	10,694	11,023	10,786	11,199	11,605	11,480	11,416	11,375	11,396
5216	1,976	3,920	6,601	7,198	8,310	8,942	9,198	8,960	9,429	10,168	10,722	10,314	10,655	10,538	10,822	10,464	10,578	11,008	10,755	11,146	10,974	11,427	11,338	11,310	11,311	11,364	11,433
5632	2,034	3,996	6,180	7,169	7,800	9,051	8,528	9,295	9,440	9,765	10,740	10,183	10,998	10,577	10,521	10,906	10,524	10,999	10,797	11,250	11,160	11,654	11,602	11,166	11,244	11,355	11,518

K = 3104

GFlops

ACTUAL FASTEST KERNELS

AMD

N

M \ N	16	32	64	112	176	256	352	464	592	736	896	1072	1264	1472	1696	1936	2192	2464	2752	3056	3376	3712	4064	4432	4816	5216	5632
16	3	3	3	3	3	3	3	3	1	3	1	7	11	26	1	26	1	26	26	18	18	14	18	15	15	1	15
32	3	3	3	3	3	3	3	3	3	3	1	7	1	18	1	18	1	18	14	18	18	14	18	15	15	29	15
64	3	3	3	3	3	3	3	12	7	12	7	12	12	14	23	30	12	0	30	30	23	30	30	0	30	5	5
112	8	3	3	3	3	1	12	23	10	0	28	25	25	28	28	28	25	22	5	5	28	5	5	28	5	28	28
176	3	3	3	3	23	4	23	13	28	25	25	28	28	28	5	5	28	5	28	28	5	28	5	5	6	6	5
256	8	3	3	1	26	14	9	10	17	22	22	17	22	5	22	24	22	5	24	24	5	6	6	5	24	5	24
352	8	3	3	4	4	2	25	28	28	25	5	28	5	28	28	5	5	6	5	5	5	5	20	24	27	27	5
464	3	3	1	7	4	10	28	28	30	5	28	28	5	5	5	20	5	5	5	20	24	5	27	24	5	5	24
592	3	3	7	4	25	25	28	25	30	5	28	28	28	20	5	5	5	5	24	27	5	5	20	24	27	24	20
736	3	3	4	2	25	28	25	5	5	28	28	28	20	5	5	20	24	5	5	24	24	24	27	20	5	27	24
896	11	1	4	2	17	2	21	17	17	28	28	6	5	5	24	24	27	24	24	24	27	24	6	27	24	6	27
1072	8	1	4	25	28	25	25	28	28	28	6	5	5	5	27	5	5	5	5	24	5	6	24	27	24	6	6
1264	11	1	4	25	28	28	5	5	28	6	5	5	5	27	5	20	24	5	20	24	24	27	27	20	27	20	27
1472	11	18	4	28	25	5	25	5	6	5	28	5	27	5	24	27	24	5	6	20	6	24	27	6	6	6	6
1696	1	1	7	10	25	28	25	5	28	28	24	27	5	24	5	20	24	24	27	24	27	24	27	6	20	27	27
1936	11	1	17	28	28	5	5	20	5	20	24	5	20	27	20	27	24	27	24	27	24	24	27	27	24	6	27
2192	1	26	12	25	25	28	5	5	5	24	27	5	24	24	24	24	6	16	24	27	24	6	27	27	27	27	27
2464	26	26	17	28	5	5	28	5	5	27	24	5	5	5	5	27	6	24	27	20	24	6	27	27	27	27	27
2752	26	26	28	5	28	28	28	5	24	5	5	5	20	24	27	24	24	27	24	24	24	27	27	20	20	20	27
3056	18	26	22	28	28	24	5	20	27	24	5	24	24	20	24	27	27	20	24	27	16	27	27	27	27	27	27
3376	11	11	28	25	5	5	5	5	5	6	27	5	24	16	27	24	24	24	24	6	24	27	27	27	27	27	27
3712	3	11	22	5	28	6	5	27	5	24	24	24	27	24	24	24	24	24	27	27	27	27	27	27	27	27	27
4064	18	18	22	5	5	6	20	27	20	27	20	24	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27
4432	8	15	17	28	5	5	24	24	24	20	27	27	20	16	16	27	27	27	16	27	27	27	27	27	27	27	27
4816	26	15	22	5	28	28	5	5	27	5	24	24	27	16	24	20	27	27	16	27	27	27	27	27	27	27	27
5216	11	19	5	28	5	28	27	5	24	27	20	24	20	27	27	16	27	27	20	27	27	27	27	27	27	27	27
5632	19	19	22	24	5	5	24	24	6	24	27	24	27	24	27	27	27	27	27	27	27	27	27	27	27	27	27

KERNEL SELECTION

- Benchmarkmarking → Raw Data
  - All Problem Sizes
  - All Kernels
  - GPU
- Analysis → Recursive Size Splittings
- Write Kernel Selection C Code

```
// Exact Sizes
if (sizeI==101 && sizeJ==202 && sizeK==303)
        return function_ptr_0;

// Range Sizes - Recursive Size Splitting
if (sizeI < threshold_I_0) {
        if (sizeJ < threshold_J_0) {
                if (sizeK < threshold_K_0)
                        return function_ptr_2;
                if (sizeK < threshold_K_1)
                        return function_ptr_3;
                // more K thresholds
                return function_ptr_4;
        }
        if (sizeJ < threshold_J_1) {
                // more K thresholds
        }
        // more J, K thresholds
}
if (sizeI < threshold_I_1) {
        // more J, K thresholds
}
// more I, J, K thresholds
```

PERFORMANCE EXPERIMENTS

AMD

- Set of 100 kernels
  - ThreadTile = 8x8, 8x4, 8x2, 4x8, 4x4, 4x2, 2x8, 2x4, 2x2
  - WorkGroup = 16x16x1, 16x8x2, 8x16x2, 8x8x4
  - GlobalSplitU = 1, 2, 4, 6, 8
  - DepthU = 8
  - VectorWidth = max
  - PrefetchGlobalRead = True
  - PrefetchLocalRead = True
  - WorkGroupMapping = 8
- Benchmark kernels against 2D range of sizes for sgemm NT
  - M = 16 – 5632; N = 16 – 5632; K = 3104
- Analyze performance and kernel properties for each data point.

PERFORMANCE

AMD

M \ N	16	32	64	112	176	256	352	464	592	736	896	1072	1264	1472	1696	1936	2192	2464	2752	3056	3376	3712	4064	4432	4816	5216	5632
16	29	62	118	205	314	448	596	735	890	1,024	1,131	1,204	1,401	1,502	1,562	1,625	1,646	1,750	1,754	1,823	1,833	1,887	1,937	1,890	1,976	1,977	2,036
32	60	116	238	406	615	880	1,137	1,424	1,722	2,006	2,271	2,304	2,628	2,877	2,953	3,120	3,171	3,365	3,452	3,617	3,619	3,654	3,800	3,710	3,848	3,896	3,992
64	121	236	452	774	1,168	1,635	2,030	2,402	2,785	3,173	3,539	3,625	4,206	4,506	4,723	4,959	4,936	5,146	5,483	5,846	5,644	5,979	6,404	5,869	6,251	6,449	6,510
112	206	409	790	1,270	1,831	2,365	2,758	3,275	3,590	3,989	4,353	4,398	5,002	5,487	5,422	6,033	5,625	5,791	6,362	6,879	6,652	6,891	7,436	7,058	7,203	7,016	7,458
176	316	633	1,201	1,848	2,404	3,178	3,457	4,012	4,506	4,695	5,479	5,397	6,223	5,894	6,204	6,908	6,755	7,237	7,001	7,332	7,864	7,630	8,156	7,868	7,704	8,343	8,023
256	456	878	1,575	2,342	3,065	4,021	4,271	4,847	5,052	5,810	5,780	5,968	6,579	6,898	6,811	7,593	7,095	7,458	7,936	8,597	8,012	8,558	9,211	8,357	8,761	8,387	8,835
352	604	1,186	2,141	2,971	3,566	4,566	4,720	5,458	5,924	6,001	6,555	6,683	7,363	7,298	7,224	7,899	7,811	7,934	7,886	8,573	8,636	8,741	9,013	8,483	8,534	9,188	8,854
464	755	1,447	2,489	3,340	4,113	5,100	5,480	5,877	5,810	6,906	7,229	7,075	7,789	7,932	7,927	8,217	8,232	8,403	8,562	8,950	8,526	8,708	9,437	8,882	9,007	8,905	9,378
592	947	1,711	2,892	3,749	4,510	5,561	6,022	5,942	6,393	7,288	7,512	7,550	7,734	8,013	8,120	8,403	8,677	8,430	8,848	9,118	8,947	8,898	9,573	9,414	9,861	9,461	9,707
736	1,073	2,018	3,415	4,254	4,966	6,241	6,099	6,967	7,254	7,666	7,913	8,111	8,476	8,693	8,424	9,010	8,796	9,114	9,079	9,678	9,715	9,801	10,293	9,534	9,505	10,182	9,853
896	1,228	2,225	3,755	4,450	5,394	6,442	6,447	6,794	6,950	7,527	7,974	8,695	8,841	8,860	9,428	9,395	9,785	9,516	9,655	9,758	10,327	10,173	10,338	10,442	10,216	10,456	10,786
1072	1,304	2,380	3,895	4,616	5,536	6,392	6,786	7,242	7,635	8,173	8,763	8,442	9,037	9,050	9,180	9,259	9,446	9,620	9,836	10,038	9,745	10,054	10,334	10,281	10,016	10,399	10,338
1264	1,427	2,716	4,490	5,222	6,367	7,212	7,401	7,782	7,755	8,474	9,031	9,012	9,125	9,373	9,506	9,739	9,929	9,614	10,071	10,300	10,133	10,140	11,079	10,544	11,055	10,580	11,158
1472	1,509	2,924	4,700	5,562	6,147	7,643	7,476	7,889	7,948	8,619	8,981	8,968	9,299	9,585	9,790	9,854	9,897	9,804	10,213	10,185	10,124	10,565	10,824	10,323	10,434	10,564	10,728
1696	1,608	3,128	4,896	5,594	6,562	7,673	7,503	8,015	8,196	8,409	9,484	9,100	9,521	9,792	9,641	9,605	9,943	9,926	10,056	10,052	10,381	10,304	10,753	10,358	10,234	10,841	10,593
1936	1,673	3,162	5,085	6,223	6,961	8,097	7,939	8,244	8,373	9,018	9,442	9,245	9,734	9,894	9,617	10,001	10,076	10,313	10,328	10,756	10,301	10,183	10,874	10,575	10,411	10,514	11,049
2192	1,666	3,291	5,152	6,069	7,078	8,025	7,870	8,285	8,692	8,773	9,876	9,419	9,929	9,894	9,946	10,081	9,841	10,083	10,276	10,515	10,485	10,543	10,970	10,806	10,726	10,663	10,651
2464	1,746	3,438	5,511	6,385	7,275	8,370	8,098	8,424	8,438	9,103	9,541	9,608	9,661	9,778	9,924	10,247	10,056	10,339	10,607	10,577	10,396	10,549	11,076	11,013	11,005	11,025	11,065
2752	1,805	3,501	5,703	6,353	7,109	8,311	8,062	8,530	8,825	9,046	9,846	9,765	10,032	10,185	10,053	10,301	10,260	10,617	10,726	10,485	10,647	11,294	11,298	10,608	10,669	10,732	10,868
3056	1,848	3,623	5,879	6,856	7,724	8,901	8,576	8,932	9,107	9,681	9,918	10,008	10,310	10,268	10,040	10,734	10,520	10,516	10,508	11,474	10,751	11,497	11,552	10,864	11,024	11,207	11,393
3376	1,836	3,570	5,910	6,907	7,934	8,733	8,671	8,510	8,930	9,739	10,468	9,701	10,125	10,145	10,398	10,296	10,481	10,394	10,669	10,722	10,744	10,801	10,989	11,199	10,775	11,037	11,290
3712	1,890	3,701	6,048	6,773	7,709	8,632	8,643	8,630	8,760	9,770	10,161	9,957	10,041	10,525	10,275	10,136	10,441	10,462	11,227	11,372	10,682	10,935	11,159	11,420	11,065	11,343	11,700
4064	1,948	3,824	6,511	7,413	8,252	9,440	8,982	9,380	9,553	10,282	10,555	10,289	11,005	10,828	10,748	10,806	10,915	11,081	11,257	11,483	10,935	11,228	11,521	11,210	11,548	11,328	11,685
4432	1,938	3,700	6,049	7,225	7,945	9,047	8,473	8,873	9,381	9,555	10,561	10,279	10,599	10,338	10,382	10,563	10,819	11,061	10,643	10,856	11,197	11,554	11,277	11,100	11,480	11,371	11,310
4816	1,966	3,819	6,334	7,244	7,879	9,041	8,608	9,070	9,868	9,534	10,245	9,994	11,052	10,466	10,211	10,420	10,701	11,044	10,694	11,023	10,786	11,199	11,605	11,480	11,416	11,375	11,396
5216	1,976	3,920	6,601	7,198	8,310	8,942	9,198	8,960	9,429	10,168	10,722	10,314	10,655	10,538	10,822	10,464	10,578	11,008	10,755	11,146	10,974	11,427	11,338	11,310	11,311	11,364	11,433
5632	2,034	3,996	6,180	7,169	7,800	9,051	8,528	9,295	9,440	9,765	10,740	10,183	10,998	10,577	10,521	10,906	10,524	10,999	10,797	11,250	11,160	11,654	11,602	11,166	11,244	11,355	11,513

K = 3104

GFlops

WINNERS

AMD

N

M

	16	32	64	112	176	256	352	464	592	736	896	1072	1264	1472	1696	1936	2192	2464	2752	3056	3376	3712	4064	4432	4816	5216	5632
16	3	3	3	3	3	3	3	3	1	3	1	7	11	26	1	26	1	26	26	18	18	14	18	15	15	1	15
32	3	3	3	3	3	3	3	3	3	3	1	7	1	18	1	18	1	18	14	18	18	14	18	15	15	29	15
64	3	3	3	3	3	3	3	12	7	12	7	12	12	14	23	30	12	0	30	30	23	30	30	0	30	5	5
112	8	3	3	3	3	1	12	23	10	0	28	25	25	28	28	28	25	22	5	5	28	5	5	28	5	28	28
176	3	3	3	3	23	4	23	13	28	25	25	28	28	28	5	5	28	5	28	28	5	28	5	5	6	6	5
256	8	3	3	1	26	14	9	10	17	22	22	17	22	5	22	24	22	5	24	24	5	6	6	5	24	5	24
352	8	3	3	4	4	2	25	28	28	25	5	28	5	28	28	5	5	6	5	5	5	5	20	24	27	27	5
464	3	3	1	7	4	10	28	28	30	5	28	28	5	5	5	20	5	5	5	20	24	5	27	24	5	5	24
592	3	3	7	4	25	25	28	25	30	5	28	28	28	20	5	5	5	5	24	27	5	5	20	24	27	24	20
736	3	3	4	2	25	28	25	5	5	28	28	28	20	5	5	20	24	5	5	24	24	24	27	20	5	27	24
896	11	1	4	2	17	2	21	17	17	28	28	6	5	5	24	24	27	24	24	24	27	24	6	27	24	6	27
1072	8	1	4	25	28	25	25	28	28	28	6	5	5	5	27	5	5	5	5	24	5	6	24	27	24	6	6
1264	11	1	4	25	28	28	5	5	28	6	5	5	5	27	5	20	24	5	20	24	24	27	27	20	27	20	27
1472	11	18	4	28	25	5	25	5	6	5	28	5	27	5	24	27	24	5	6	20	6	24	27	6	6	6	6
1696	1	1	7	10	25	28	25	5	28	28	24	27	5	24	5	20	24	24	27	24	27	24	27	6	20	27	27
1936	11	1	17	28	28	5	5	20	5	20	24	5	20	27	20	27	24	27	24	27	24	24	27	27	24	6	27
2192	1	26	12	25	25	28	5	5	5	24	27	5	24	24	24	24	6	16	24	27	24	6	27	27	27	27	27
2464	26	26	17	28	5	5	28	5	5	27	24	5	5	5	5	27	6	24	27	20	24	6	27	27	27	27	27
2752	26	26	28	5	28	28	28	5	24	5	5	5	20	24	27	24	24	27	24	24	24	27	27	20	20	20	27
3056	18	26	22	28	28	24	5	20	27	24	5	24	24	20	24	27	27	20	24	27	16	27	27	27	27	27	27
3376	11	11	28	25	5	5	5	5	5	6	27	5	24	16	27	24	24	24	24	6	24	27	27	27	27	27	27
3712	3	11	22	5	28	6	5	27	5	24	24	24	27	24	24	24	24	24	27	27	27	27	27	27	27	27	27
4064	18	18	22	5	5	6	20	27	20	27	20	24	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27
4432	8	15	17	28	5	5	24	24	24	20	27	27	20	16	16	27	27	27	16	27	27	27	27	27	27	27	27
4816	26	15	22	5	28	28	5	5	27	5	24	24	27	16	24	20	27	27	16	27	27	27	27	27	27	27	27
5216	11	19	5	28	5	28	27	5	24	27	20	24	20	27	27	16	27	27	20	27	27	27	27	27	27	27	27
5632	19	19	22	24	5	5	24	24	6	24	27	24	27	24	27	27	27	27	27	27	27	27	27	27	27	27	27

- Neighboring kernels may be similar or different.
- 1st and 2nd fastest kernels may be similar or different (parameters and performance).

FLOPS/BYTE OF TILE

Flops/Byte = 2*MT0*MT1 / (MT0+MT1)/4

For example 128x128 tile:

2*128*128/(128+128)/4 = 32 F/B

2* 64* 64/(64+ 64)/4 = 16 F/B

M \ N	16	32	64	112	176	256	352	464	592	736	896	1072	1264	1472	1696	1936	2192	2464	2752	3056	3376	3712	4064	4432	4816	5216	5632
16	8	8	8	8	8	8	8	8	6	8	6	6	6	4	6	4	6	4	4	4	4	4	4	2	2	6	2
32	8	8	8	8	8	8	8	8	8	8	8	11	8	8	8	8	8	8	11	8	8	11	8	8	8	11	8
64	8	8	8	8	8	8	8	11	11	11	11	11	11	11	11	13	11	11	13	13	11	13	13	11	13	16	16
112	8	8	8	8	8	8	11	11	13	11	16	16	16	16	16	16	16	13	16	16	16	16	16	16	16	16	16
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1264	8	8	11	16	16	16	16	16	16	21	16	16	16	32	16	32	16	16	32	16	16	32	32	32	32	32	32
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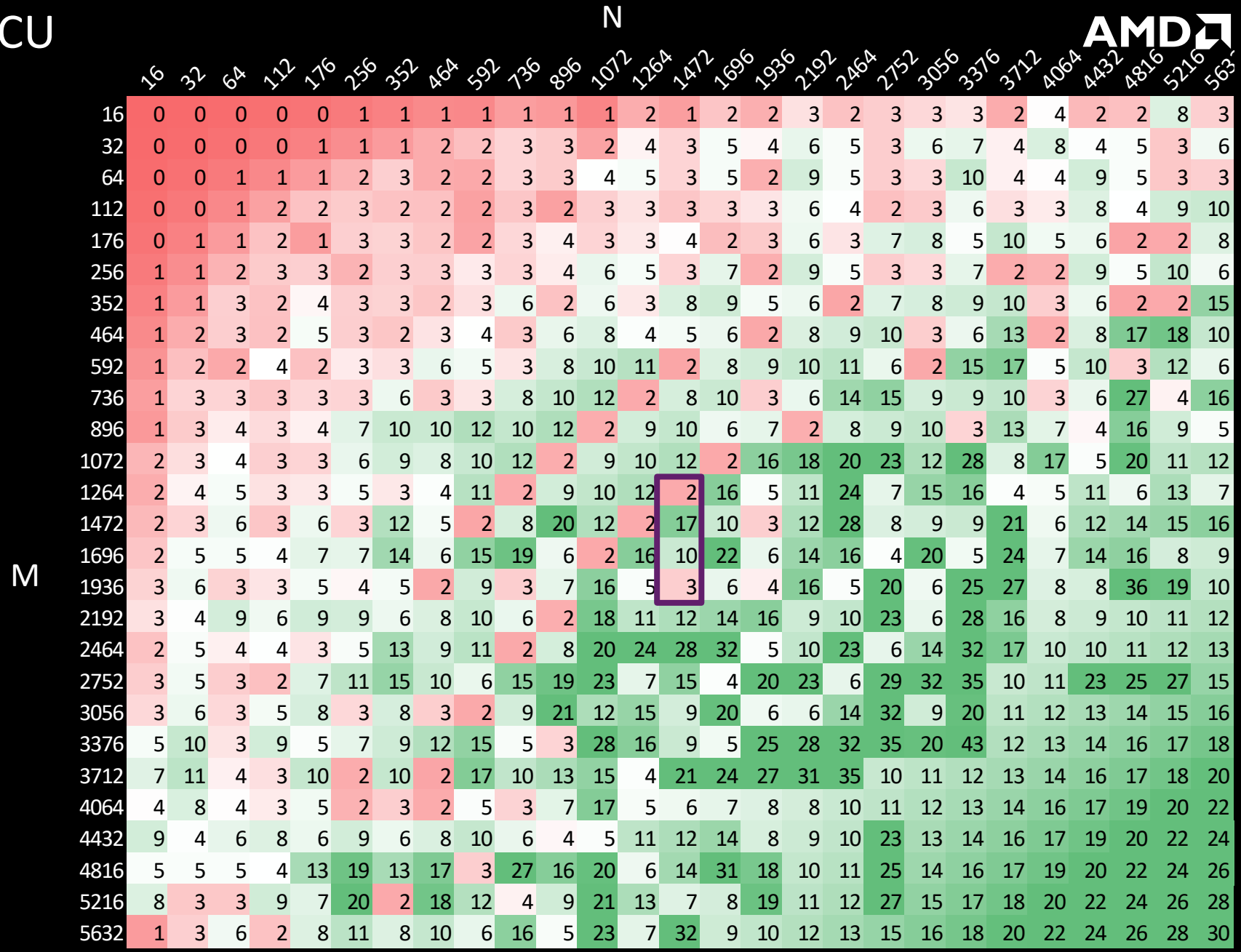
5216	8	11	16	16	16	16	32	16	16	32	32	16	32	32	32	21	32	32	32	32	32	32	32	32	32	32	32
5632	11	11	13	16	16	16	16	16	21	16	32	16	32	16	32	32	32	32	32	32	32	32	32	32	32	32	32

TOTAL WORKGROUPS / 64CU



M \ N	16	32	64	112	176	256	352	464	592	736	896	1072	1264	1472	1696	1936	2192	2464	2752	3056	3376	3712	4064	4432	4816	5216	563
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464	1	2	3	2	5	3	2	3	4	3	6	8	4	5	6	2	8	9	10	3	6	13	2	8	17	18	10
592	1	2	2	4	2	3	3	6	5	3	8	10	11	2	8	9	10	11	6	2	15	17	5	10	3	12	6
736	1	3	3	3	3	3	6	3	3	8	10	12	2	8	10	3	6	14	15	9	9	10	3	6	27	4	16
896	1	3	4	3	4	7	10	10	12	10	12	2	9	10	6	7	2	8	9	10	3	13	7	4	16	9	5
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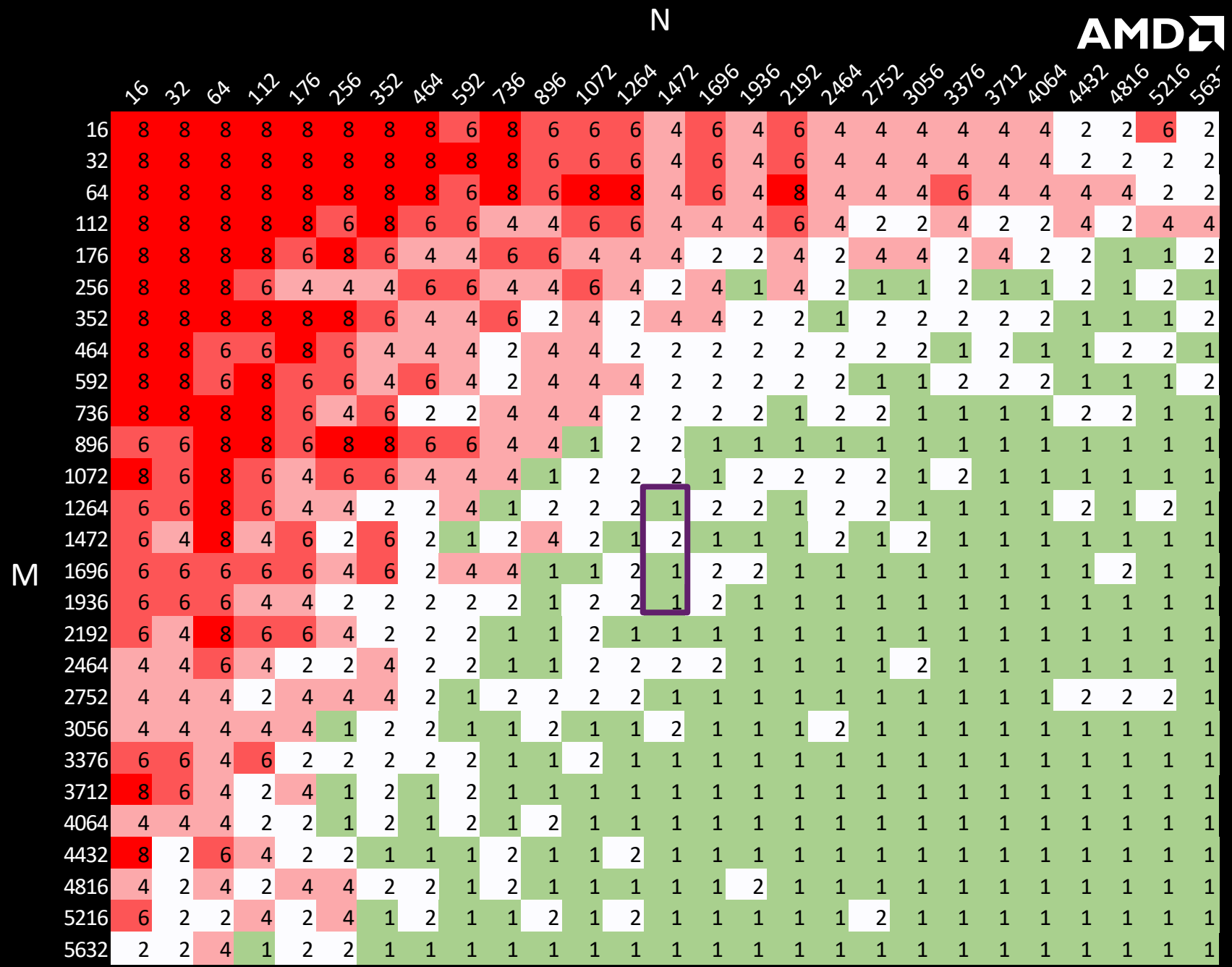
5632	1	3	6	2	8	11	8	10	6	16	5	23	7	32	9	10	12	13	15	16	18	20	22	24	26	28	30

GLOBAL SPLIT U



KERNEL SELECTION

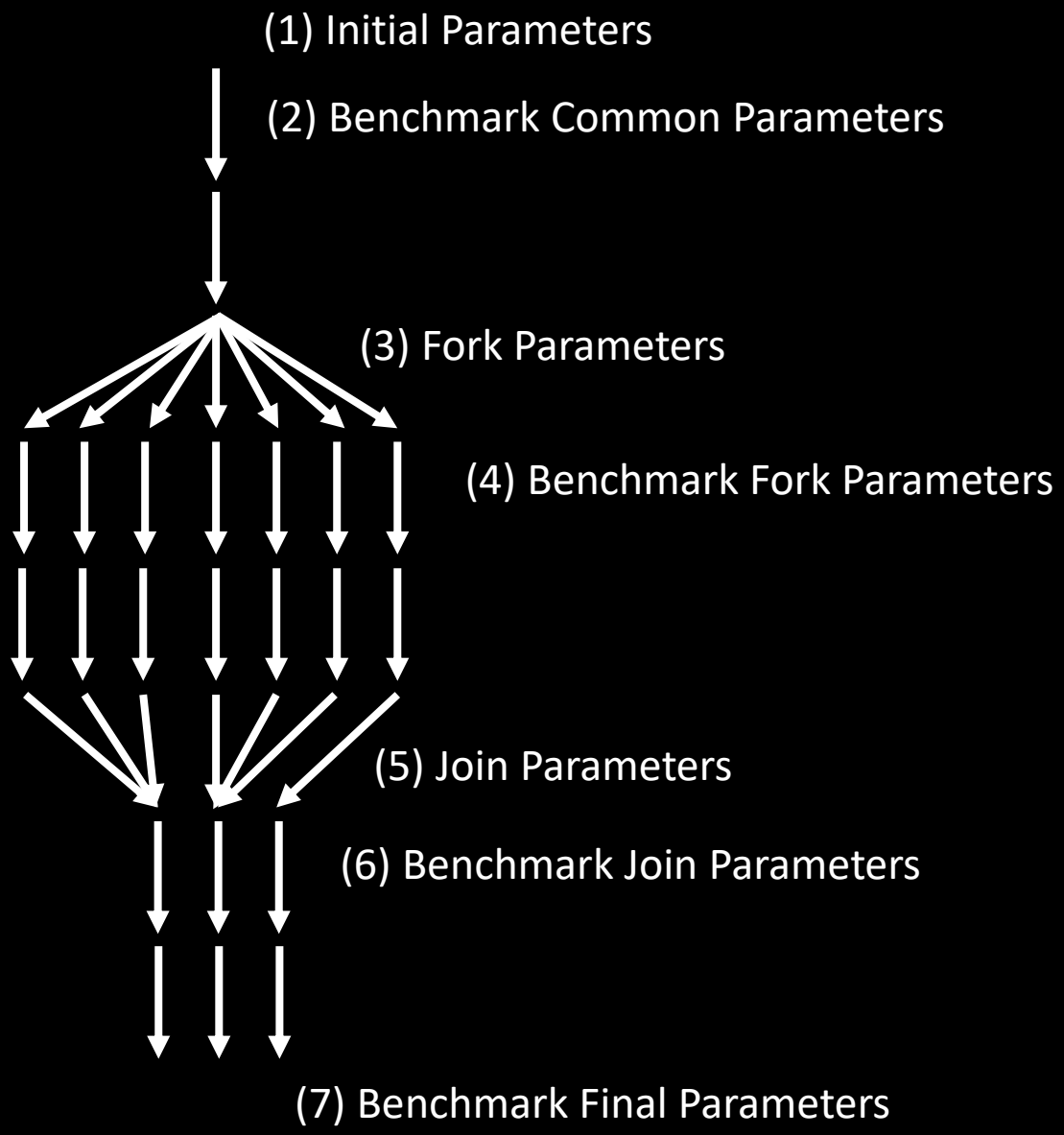
- Benchmarking → Raw Data
  - All Problem Sizes
  - All Kernels
  - GPU
- Analysis → Recursive Size Splittings
- Write Kernel Selection C Code

Too Time Intensive!

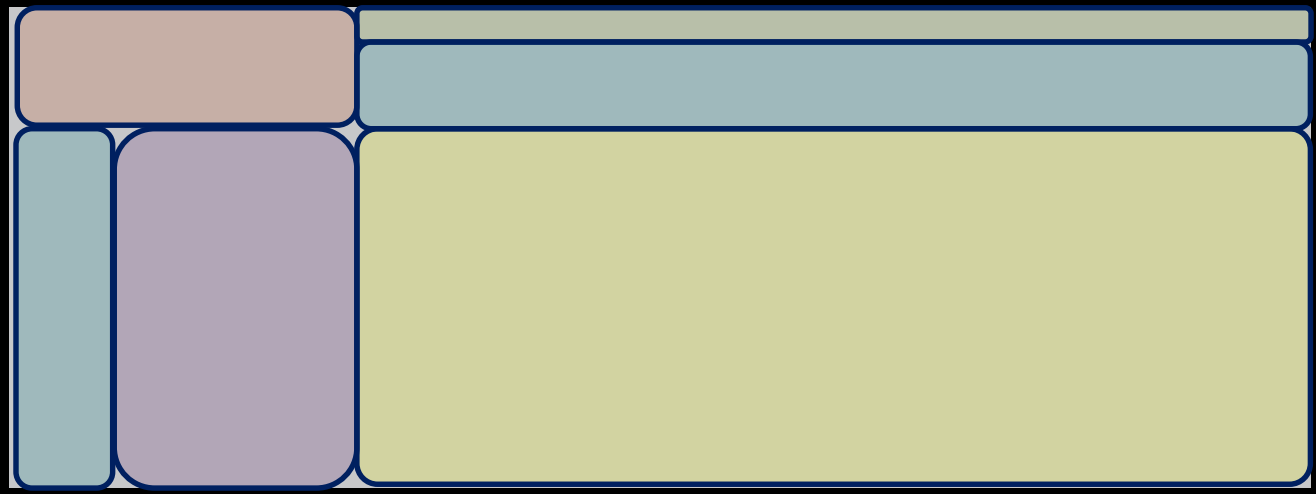
```
// Exact Sizes
                                    zeJ==202 && sizeK==303)
                                    tion_ptr_0;

                                    ursive Size Splitting
if (sizeI < threshold_I_0) {
        if (sizeJ < threshold_J_0) {
                if (sizeK < threshold_K_0)
                        return function_ptr_2;
                if (sizeK < threshold_K_1)
                        return function_ptr_3;
                // more K thresholds
                return function_ptr_4;
        }
        if (sizeJ < threshold_J_1) {
                // more K thresholds
        }
        // more J, K thresholds
}
if (sizeI < threshold_I_1) {
        // more J, K thresholds
}
// more I, J, K thresholds
```

AUTO-TUNING



- Breakup into problem size groups.
  - small/skinny tiles for small/skinny problem sizes
  - large tiles for large problem sizes
- Steps 1-6 create a set of candidates which will be benchmarked against that size group.
- User can limit number of kernels used in library.



CONCLUSIONS

- Fast GEMM on GPUs isn't just one kernel, its many.
- Different problems sizes want different kernels.
- More kernels and more precise mappings will improve performance.

N

M	16	32	64	112	176	256	352	464	592	736	896	1072	1264	1472	1696	1936	2192	2464	2752	3056	3376	3712	4064	4432	4816	5216	5632
16	3	3	3	3	3	3	3	3	1	3	1	7	11	26	1	26	1	26	26	18	18	14	18	15	15	1	15
32	3	3	3	3	3	3	3	3	3	3	1	7	1	18	1	18	1	18	14	18	18	14	18	15	15	29	15
64	3	3	3	3	3	3	3	12	7	12	7	12	12	14	23	30	12	0	30	30	23	30	30	0	30	5	5
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176	3	3	3	3	23	4	23	13	28	25	25	28	28	28	5	5	28	5	28	28	5	28	5	5	6	6	5
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736	3	3	4	2	25	28	25	5	5	28	28	28	20	5	5	20	24	5	5	24	24	24	27	20	5	27	24
896	11	1	4	2	17	2	21	17	17	28	28	6	5	5	24	24	27	24	24	24	27	24	6	27	24	6	27
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1696	1	1	7	10	25	28	25	5	28	28	24	27	5	24	5	20	24	24	27	24	27	24	27	6	20	27	27
1936	11	1	17	28	28	5	5	20	5	20	24	5	20	27	20	27	24	27	24	27	24	24	27	27	24	6	27
2192	1	26	12	25	25	28	5	5	5	24	27	5	24	24	24	24	6	16	24	27	24	6	27	27	27	27	27
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3056	18	26	22	28	28	24	5	20	27	24	5	24	24	20	24	27	27	20	24	27	16	27	27	27	27	27	27
3376	11	11	28	25	5	5	5	5	5	6	27	5	24	16	27	24	24	24	24	6	24	27	27	27	27	27	27
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5632	19	19	22	24	5	5	24	24	6	24	27	24	27	24	27	27	27	27	27	27	27	27	27	27	27	27	27

Use GPUs

Thank you